Calculus and Laplace Transforms

Answer all the Questions

Q. No.	Question Description	Marks
1	Discuss the continuity of the function at the origin $f(x,y) = \begin{cases} \frac{2x(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$	10
2	A manufacturer's production modeled by Cobb-Douglas function $f(x,y) = 100\, x^{\frac{3}{4}} y^{\frac{1}{4}}$ where x represents the units of labor and y represents the units of capital. Each labor unit cost costs 150 rupees and each capital unit costs 250 rupees. The total expenses for labor and capital cannot exceed 50,000 rupees. Will the maximum production level exceed 16,000 units?	10
3	Reverse the order of integration, and evaluate the integral. $\int_0^{\frac{1}{16}} \int_{y^{\frac{1}{4}}}^{\frac{1}{2}} \cos(16\pi x^5)\, dx\, dy$	10
4	Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.	10
5	If $F = (2x^2 - 3z)i - 2xyj - 4xk$, evaluate a) $\int_V \nabla . F\, dV$ b) $\int_V \nabla \times F\, dV$ Where V is the closed region bounded by $x = 0, y = 0, z = 0$, $2x + 2y + z = 4$.	10

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