Calculus and Laplace Transforms

Q. No.	Question Description	Marks

PART - A – (3 x 10 = 30 Marks)

1 **(a)** Let $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^3+y^3}, & for\ f(x,y) \neq (0,0) \\ 15, & for\ f(x,y) = (0,0) \end{cases}$ **10**
Check the continuity at $(1,1)$ and at the origin.

OR

(b) A cylindrical hole of radius a is bored through a sphere of radius b. Find the volume **10**
of the remaining solid.

2 **(a)** If $\phi = \frac{3}{8}xyz$ find $\int_S \phi N\ dS$ where S is the surface of the cylinder $x^2 + y^2 = 16$ **10**
included in the first octant between $z = 0$ and $z = 5$.

OR

(b) By transforming into triple integral, compute $\oint_S (ax^2 + by^2 + cz^2)dS$ over the **10**
sphere cantered at origin and radius is 1.

3 **(a)** Find the equation of the curve passes through $(1,1)$, whose differential equation is **10**
$$(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$$

OR

(b) Find the Laplace transform of **10**

$$f(t) = \sinh ct \int_0^t e^{au} \sinh bu\ du$$

Part - B – (2 x 10 = 20 Marks)

4 Raju is planning to build an automobile start up, the annual estimation cost to run **10**
the start up for its labor and equipment cost (in Lakhs) can be modeled by

$$f(x,y) = 2x^2 + 3y^2 - 15x - 20y + 4xy + 39$$

Where x is the amount spent per year on labor and y is the amount spent per year on equipment (both in Lakhs). Find the values of x and y that minimize the annual labor and equipment cost. What is this cost?

5   Find the general solution of the following differential equation   10

$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = e^{ax} + e^{bx}$$

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